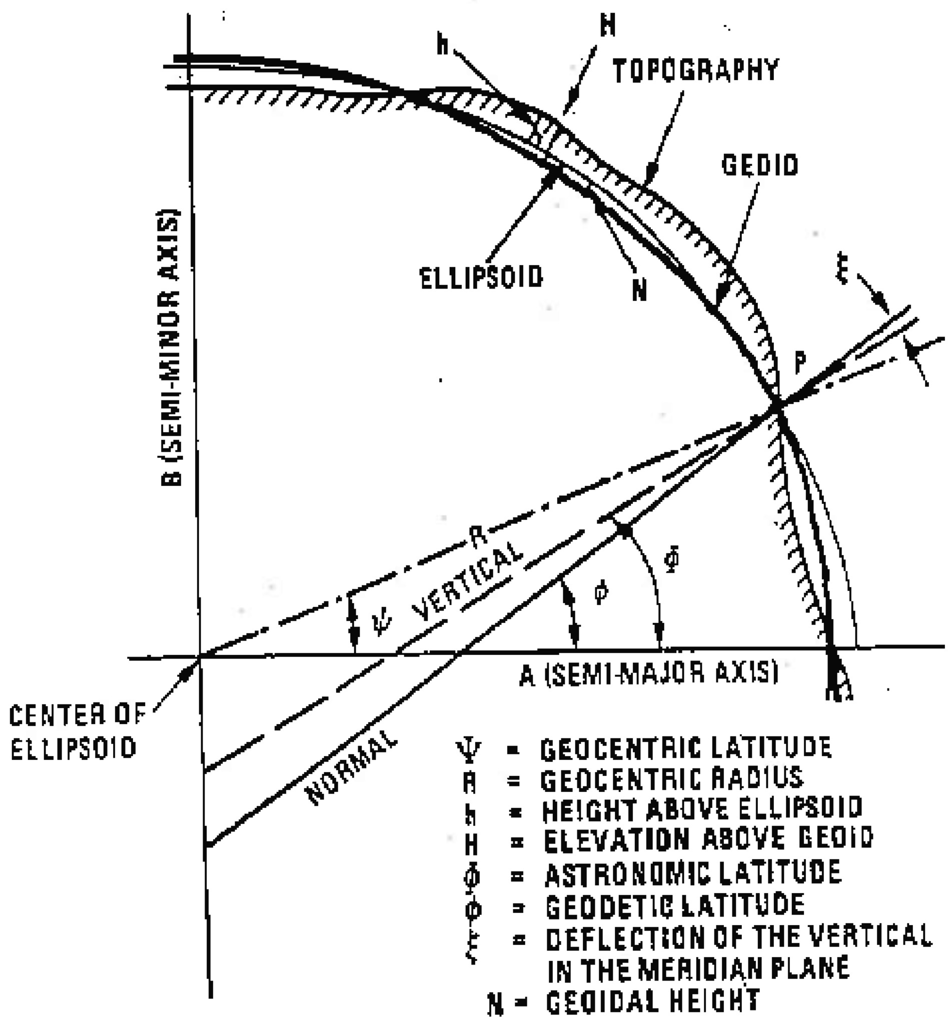
USING STATE PLANE COORDINATES



City of Ann Arbor Geodetic Control Manual

EXAMPLE SURVEY USING STATE PLANE COORDINATES

This subsection gives information about an example survey using state plane coordinates. It poses a sample problem and describes the solution. Although this particular example is in metric units, the procedure will be the same for surveys based in international feet.

Sample Problem

A second order traverse was run from GPS control station 10000 and closed at control station 10002. Three intermediate traverse stations (T1, T2, and T3) were established. At Station T2 side shots were taken to the four corners of Lot A.

Given the control information data and the reduced field note information of the traverse, compute the adjusted NAD 83 state plane coordinates (Michigan South Zone 2113) of the intermediate traverse stations and property corners of Lot A. The reduced field notes use the following assumptions:

- The traverse was oriented with respect to grid azimuth at the starting control station.
- The observed directions have been adjusted for closure.

Use the following control information data:

- **GPS Control Station 10000**

  N_1 = 82930.962 (northing in meters)
  E_1 = 4049750.024 (easting in meters)
  H_1 = 280.170 (elevation in meters)
  K_1 = 0.99996880 (grid scale factor)
  G_1 = -34.164 (geoidal height in meters)
  Grid azimuth to station 10001 = 42° 15' 08"

- **GPS Control Station 10002**

  N_2 = 82681.549 (northing in meters)
  E_2 = 4051783.560 (easting in meters)
  H_2 = 261.750 (elevation in meters)
  K_2 = 0.99996930 (grid scale factor)
  G_2 = -34.189 (geoidal height in meters)
  Grid azimuth to station 10001 = 178° 48' 42"

Table 1.

STATION		GRID AZIMUTH	GROUND DISTANCE HORIZONTAL (METERS)	ELEVATION (METERS)
OCC.	OBS.			
10000				280.170
	10001	42-15-08		
	T1	48-56-13	527.714	265.335
T1				
	T2	120-26-05	403.546	262.159
T2				
	Cor1	225-00-00	21.555	260.635
	Cor2	75-57-49	62.842	259.995
	Cor3	111-48-06	82.077	257.557
	Cor4	146-18-36	54.954	262.891
	T3	88-21-30	620.840	257.785
T3				
	10002	121-31-53	782.877	261.750
10002				
	10003	178-48-42		

Solution Steps

1. Draw a sketch (see Figure 1 for an example).

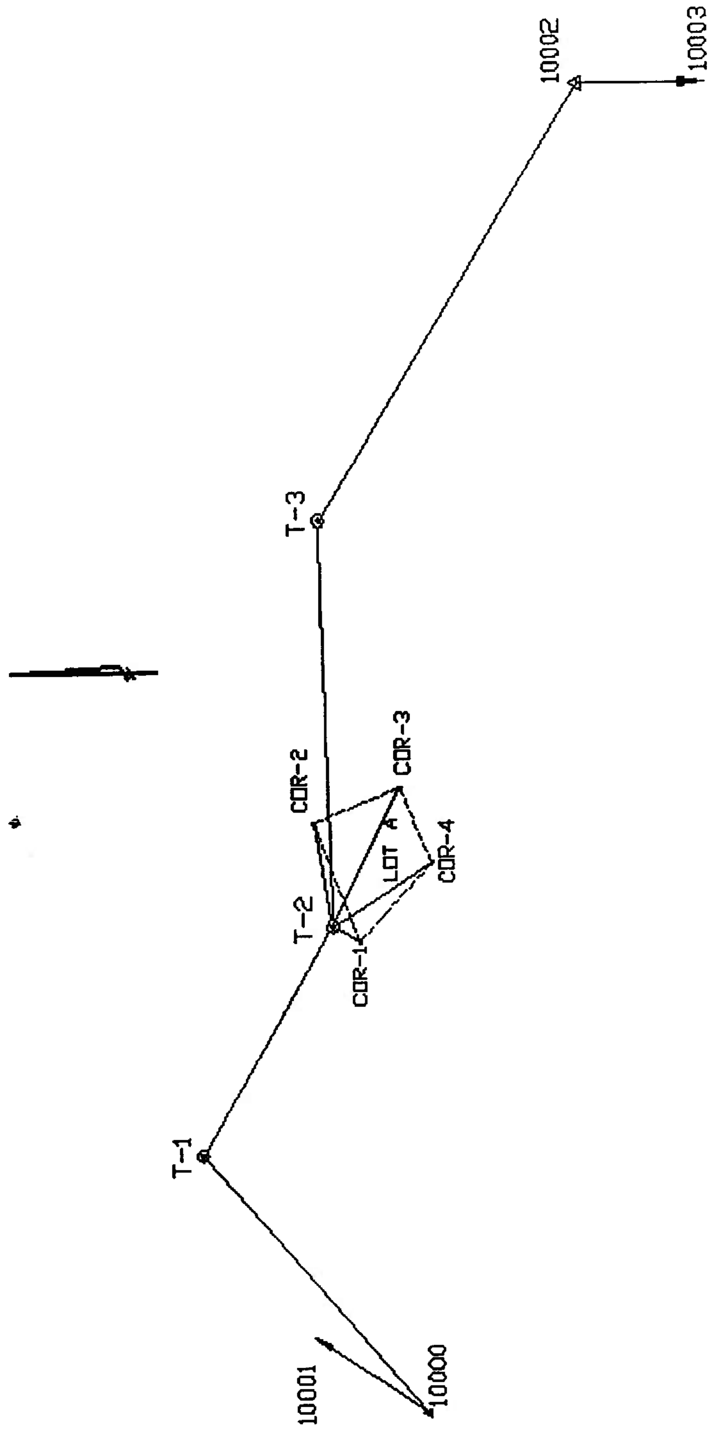


Figure 1.

2. Analyze the grid scale factor for the project. A mean of the published point grid scale factors of the points maybe adequate for all lines in the project.

   Michigan South Zone uses the Lambert Projection for the Lambert grid coordinates. The formula for computing K, the grid scale factor, at any given point is as follows:

$$K = \frac{\sqrt{(1-e^2\sin^2\Phi)}\, R\sin\Phi_0}{a\cos\Phi}$$

Where		
	e^2	= eccentricity squared of the GPS 80 ellipsoid
		= 0.0066943800229034
	R	= mapping radius at latitude ϕ
	ϕ_0	= latitude, the true projection origin = 42.8850151357
	a	= semi-major axis of the GRS 80 ellipsoid = 6,378,137 m

   For the second order traverse, it may be sufficient to calculate the average $\bar{K}$:

$$\bar{K} = \frac{\sum_i^n K_i}{n}$$

   Where n = number of control points

   For surveys requiring better accuracy, a more accurate formula for $\bar{K}$ may be used (see pages 49 and 50 of *NOAA Manual NGS 5: State Plane Coordinate System, 1983*)

$$\bar{K} = \frac{K_1 + 4K_m + K_2}{6}$$

   Where K_m = grid scale factor at the midpoint of the line joining points 1 and 2

3. Analyze the elevation factors for the project. Observed horizontal distance must be reduced to the reference ellipsoid by applying this factor, as shown in Figure 2:

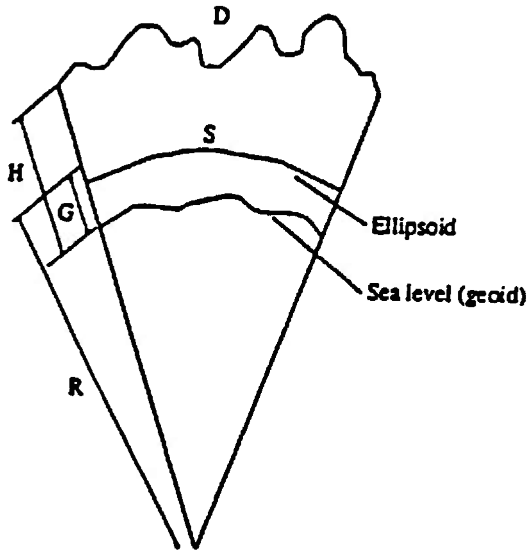


Figure 2.

By proportion:

$$\frac{S}{D} = \frac{R}{R + H + G} = C_H$$

Where
C_H = elevation factor
S = ellipsoidal distance
D = horizontal distance
R = mean radius of the Earth = 6,372,000 m
H = mean elevation
G = geoid-ellipsoid separation (geoidal height)

Note: Use +G if the ellipsoid is below the geoid; use -G if the ellipsoid is above the geoid.

In the conterminous United States, the GRS 80 (WGS84) ellipsoid is always above the geoid as shown in the sketch.

4. Compute the combined factor $C = \bar{K} \times C_H$ for each observed horizontal distance.

5. Reduce the horizontal distance to the grid by multiplying the observed horizontal distance by its corresponding combined factor.

6. Adjust the traverse using the compass rule adjustment method.

7. Compute the adjusted NAD 83 state plane coordinate of each point.

   a. Using the sketch, compute a grid scale factor $\bar{K}$:

$$\bar{K} = \frac{1}{2}(K_1 + K_2)$$

Where $\bar{K}$ = ½(0.99996880 + 0.99996930)
= 0.99996905

   b. Compute the elevation factor for each line:

$$\frac{S}{D} = \frac{R}{R + H + G} = C_H$$

Where R = 6,372,000 m
G = mean geoidal height
= $\frac{1}{2}(G_1 + G_2)$ = ½[(-34.164) + (-34.189)]
= -34.176

In the City of Ann Arbor, G averaged about –34.29 m. Compute C_H of line 10000 to T1, and call it C_{H1}:

Where H_1 = Mean elevation of 10000 and T1
= ½(280.170 + 265.335) = 272.753 m

C_{H1} = 6,372,000 m ÷ (6,372,000 + 272.753 - 34.176)
= 0.999962560

   c. Compute the combined factor $C = \bar{K} \times C_H$, for example, for line 10000 to T1and call it C_1 :

Where C_1 = $\bar{K} \times C_{H1}$ = 0.99996905 × (0.999962560)
= 0.999931611

d. Compute the grid distance by multiplying the measured horizontal distance by the combined factor. For example, for line 10000 to T1, the grid distance is 527.714 × 0.999931611 = 527.678 m.

Steps 7b to 7d can now be summarized in tabulated form as follows:

Table 2.

LINE	$\bar{K}$	C_H	C	GRID DISTANCE
10000 – T1	0.99996905	0.999962560	0.999931611	527.678
T1 – T2	0.99996905	0.999963973	0.999933024	403.519
T2 – Cor1	0.99996905	0.999964342	0.999933393	21.553
T2 – Cor2	0.99996905	0.999964392	0.999933443	62.837
T2 – Cor3	0.99996905	0.999964583	0.999933634	82.072
T2 – Cor4	0.99996905	0.999964165	0.999933216	54.950
T2 – T3	0.99996905	0.999964566	0.999933617	620.799
T3 – 10002	0.99996905	0.999964598	0.999933649	782.825

e. Adjust the traverse:

Table 3.

STATION	GRID AZIMUTH	GRID DISTANCE	LATITUDE	DEPARTURE	GRID COORDINATES NORTHING	GRID COORDINATES EASTING
10000				FIXED	82930.962	4049750.024
	48-56-13	527.678	346.626	397.862		
T1					83277.588	4050147.886
	120-26-05	403.519	-204.405	347.916		
T2					83073.183	4050495.803
	88-21-30	620.799	17.785	620.544		
T3					83090.968	4051116.347
	121-31-53	782.825	-409.391	667.244		
10002					82681.578	4051783.591
					82681.549	4051783.560

Where
ΣLatitude = -249.384
ΣDeparture = 2033.567
$N_{10002} - N_{10000}$ = -249.413
$E_{10002} - E_{10000}$ = 2033.536
dN_t = -0.0287
dE_t = -0.031
L = Σgrid distance = 2334.821

Closure correction d_c $= \sqrt{(dN_t^2 + dE_t^2)}$
$= \sqrt{[(-0.0287^2) + (-0.031^2)]}$
$= 0.042$

f. Compute corrections ΔN_i and ΔE_i to N and E respectively using the compass rule:

Where
ΔN_i = correction to N_i
ΔE_i = correction to E_i
dE_T = total closure correction of the traverse in the N coordinate
dN_T = total closure correction of the traverse in the E coordinate
L_i = distance from station $_i$ to the initial station
L = total length of the traverse
$\Delta N_i = L_i \ \Delta N_i \div L$ and $\Delta E_i = Li \ dE_T \div L$
$\Delta N_1 = (527.678)\times(-0.0287)\div 2334.821 = -0.006$
$\Delta N_2 = (931.197)\times(-0.0287)\div 2334.821 = -0.011$
$\Delta N_3 = (1551.996)\times(-0.0287)\div 2334.821 = -0.019$
$\Delta E_1 = (527.678)\times(-0.031)\div 2334.821 = -0.007$
$\Delta E_2 = (931.197)\times(-0.031)\div 2334.821 = -0.012$
$\Delta E_3 = (1551.996)\times(-0.031)\div 2334.821 = -0.021$

g. Compute the adjusted grid coordinate N_{ci}, E_{ci} of the traverse stations and lot corners:

Where
$N_{ci} = N_{ui} + \Delta N_i$
$E_{ci} = E_{ui} + \Delta E_i$

Where
N_{ui} = unadjusted northing of point $_i$
E_{ui} = unadjusted easting of point $_i$

Table 4.

STATION	**UNADJUSTED GRID COORDINATES**		**CORRECTIONS**		**ADJUSTED GRID COORDINATES**	
	N_u	**E_u**	**ΔN**	**ΔE**	**N_c**	**E_c**
T1	83277.588	4050147.886	-0.006	-0.007	83277.581	4050147.879
T2	83073.183	4050495.803	-0.011	-0.012	83073.172	4050495.790
T3	83090.968	4051116.347	-0.019	-0.021	83090.949	4051116.326

h. Compute the grid coordinates of the corners of Lot A:

Table 5.

STATION		GRID AZIMUTH	GRID DISTANCE	LATITUDE	DEPARTURE	GRID COORDINATES	
OCC.	OBS.					NORTHING	EASTING
T2						83073.171	4050495.790
	Cor 1	225-00-00	21.553	-15.240	-15.240	83057.932	4050480.550
	Cor 2	75-57-49	62.836	15.240	60.960	83073.172	4050541.513
	Cor 3	111-48-06	82.070	-30.480	76.200	83042.692	4050617.713
	Cor 4	146-18-36	54.949	-45.720	30.480	82996.971	4050648.194

i. Convert the grid coordinates (as needed) from metric units to International feet.

Using the conversion between the metric system and International feet of:
1 International foot = 0.3048 meters

Table 6.

STATION		GRID AZIMUTH	GRID DISTANCE	LATITUDE	DEPARTURE	GRID COORDINATES	
OCC.	OBS.					NORTHING	EASTING
T2						272549.774	13289028.182
	Cor 1	225-00-00	70.712	50.000	50.000	272499.777	13288978.182
	Cor 2	75-57-49	206.155	50.000	200.000	272549.777	13289178.192
	Cor 3	111-48-06	269.259	100.000	250.000	272449.777	13289428.192
	Cor 4	146-18-36	180.279	150.000	100.000	272299.774	13289528.196